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Selection of a Barley Yield Model Using Information–Theoretic Criteria

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Empirical models of crop–weed competition are integral components of bioeconomic models, which depend on predictions of the impact of weeds on crop yields to make cost-effective weed management recommendations. Selection of the best empirical model for a specific crop–weed system is not straightforward, however. We used information–theoretic criteria to identify the model that best describes barley yield based on data from barley–wild oat competition experiments conducted at three locations in Montana over 2 yr. Each experiment consisted of a complete addition series arranged as a randomized complete block design with three replications. Barley was planted at 0, 0.5, 1, and 2 times the locally recommended seeding rate. Wild oat was planted at target infestation densities of 0, 10, 40, 160, and 400 plants m^{-2}. Twenty-five candidate yield models were used to describe the data from each location and year using maximum likelihood estimation. Based on Akaike's Information Criterion (*AIC*), a second-order small-sample version of *AIC* (AIC_c), and the Bayesian Information Criterion (BIC), most data sets supported yield models with crop density (D_c), weed density (D_w), and the relative time of emergence of the two species (T) as variables, indicating that all variables affected barley yield in most locations. *AIC*, AIC_c, and *BIC* selected identical best models for all but one data set. In contrast, the Information Complexity criterion, *ICOMP*, generally selected simpler best models with fewer parameters. For data pooled over years and locations, *AIC*, AIC_c, and *BIC* strongly supported a single best model with variables D_c, D_w, T, and a functional form specifying both intraspecific and interspecific competition. *ICOMP* selected a simpler model with D_c and D_w only, and a functional form specifying interspecific, but no intraspecific, competition. The information–theoretic approach offers a rigorous, objective method for choosing crop yield and yield loss equations for bioeconomic models.

Nomenclature: Wild oat, *Avena fatua* L. AVEFA; barley, *Hordeum vulgare* L.

Key words: Crop–weed competition, bioeconomic models, model selection, information criteria, *AIC*, AIC_c, *BIC*, *ICOMP*, wild oat, barley, yield, yield loss, relative time of emergence.

Empirical models of crop–weed competition are widely used to make decisions concerning weed management. Such models are integral components of bioeconomic models, which depend on accurate predictions of the impact of weeds on crop yields to make cost-effective weed management recommendations. In general, empirical crop–weed competition models used in bioeconomic models take one of three basic forms; crop yields, or yield losses, are a function of (1) either crop or weed density alone, (2) weed and crop density, or (3) weed and/or crop density and the relative time of emergence of the crop and weed. Numerous functional specifications of the explanatory or predictor variables exist for each model type (reviewed in Willey and Heath 1969; Cousens 1985a,b; Cousens et al. 1987; Firbank and Watkinson 1990; Wagner et al. 2007). Because empirical models play such a pivotal role in the development of weed management recommendations, considerable time and effort has been expended evaluating such models to select the best model for a particular crop–weed system (e.g., Cousens et al. 1987; Jasieniuk et al. 2001; Martin et al. 1987; O'Donovan et al. 2005). Unfortunately, results of model evaluations have often been disappointing with unclear or mixed outcomes because of the limited number of models compared and the lack of well-defined statistical criteria for selecting the best model.

Model selection using information–theoretic criteria (Burnham and Anderson 1998, 2002; Taper 2004) is a relatively new branch of mathematical statistics that shows promise for crop–weed competition modeling. The model selection approach is an alternative to traditional null hypothesis testing, which is normally used to evaluate crop–weed competition models (e.g., Jasieniuk et al. 2001). The major advantages of the model selection approach are that it allows simultaneous assessment of multiple competing hypotheses (models) rather than the comparison of only two (the null and a single alternative) as is required in null hypothesis testing (reviewed in Franklin et al. 2001; Johnson and Omland 2004; Taper 2004). In addition, the statistical criteria for evaluating models are clearly defined. In the model selection approach, multiple candidate models are simultaneously and quantitatively evaluated in terms of the empirical evidence from the data for one model over another. Information criteria are the measures of empirical evidence for competing models (Burnham and Anderson 1998, 2002; Taper 2004). Because information criteria are estimators of the information lost when a model is used to approximate reality, a model is identified as "best" if it exhibits the minimum value of an information criterion in comparison with the remaining models. Further, the magnitude of differences in information criterion values, ΔIC, between models indicates the strength of evidence for one model over another (Taper 2004). Ultimately, the model identified as best represents the inference from the data and indicates the effects (i.e., model and model parameters) best supported by the data (Burnham and Anderson 1998, 2002). It has been suggested that model misspecification leading to incorrect inference from data is a major source of error in the quantification of scientific evidence (Chatfield 1995; Taper 2004). The strength of the model selection approach is that it allows rigorous evaluation of the empirical support for multiple competing hypotheses and thus significantly reduces the risk of model misidentification and incorrect inference. That said, the best model selected by information criteria will be the best of the

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candidate set of models examined. If the candidate set does not include models appropriate for the crop–weed system under study, the best model will perform poorly at describing the data and a better model might be found later. Only researchers who have knowledge of, and experience with, a crop–weed system can assess the biological suitability of models in a candidate set. The purpose of this paper is to inform weed scientists of a relatively new approach that can help them assess the statistical suitability of a set of candidate models in describing crop–weed competition data.

The model selection approach is widely used to evaluate competing models and make decisions concerning wildlife and natural resources management (Johnson and Omland 2004). However, with the exception of Wagner et al. (2007), the approach has not been applied to weed management despite its potential for selecting crop yield, or yield loss, models to be used in bioeconomic models. In this study, our goal was to evaluate multiple crop–weed competition models to identify the model that best describes barley yield based on data from field experiments conducted in Montana. Specifically, our objectives were to: (1) fit crop–weed competition models of varying complexity and functional form to data from barley–wild oat competition experiments; (2) select the best model, or models, based on the empirical evidence; and (3) provide an example of how information–theoretic criteria can be employed in the identification of models for weed management decision aids, such as bioeconomic models.

Materials and Methods

Field Experiments. Barley–wild oat competition experiments were conducted at three locations in Montana: the Arthur H. Post Research Farm at Bozeman, the Central Agricultural Research Center at Moccasin, and the Northwestern Agricultural Research Center at Kalispell. Experiments were conducted at Bozeman in 1993 and 1994, at Kalispell in 1993, and at Moccasin in 1994. Two independent experiments (R1 and R2) were conducted at Kalispell in 1993. For each location and year, experiments consisted of a complete addition series arranged as a randomized complete block design with three replications of 2.4 by 9.1 m plots. 'Gallatin' barley was planted at 0, 0.5, 1, and 2 times the locally recommended seeding rate of 6.7 g m^{-2} (160 to 200 seeds m^{-2}) at a depth of 3.8 cm and row spacing of 30.5 cm. Wild oat was sown at target infestation densities of 0, 10, 40, 160, and 400 plants m^{-2}. Weeds other than wild oat were manually removed throughout the growing season.

Sampling units consisted of two 0.25 m^2 permanent quadrats located randomly in each experimental plot. Times of emergence of barley and wild oat seedlings were monitored at Bozeman and Kalispell in 1993 and Bozeman and Moccasin in 1994 by counting seedling numbers in each quadrat 0, 4, 6, 8, 12, 14, 24, 32, or 0, 8, 10, 12, 16, 18, 28, 36 d after planting (DAP), respectively. The last sampling time was determined by the last day seedlings were observed to emerge. Percentage of seedlings emerged at each sampling date was calculated based on the cumulative total of emerged seedlings. Barley and wild oat densities were measured by counting reproductive tillers within each quadrat just prior to harvest when spikelets of both species were still intact. Barley and wild oat plants in permanent quadrats were harvested, counted, and the seed of each species separated and counted prior to shattering. In addition, barley grain yield was measured at maturity by harvesting 1.5 by 4.6 m areas at the center of each plot with a combine harvester. Seeds of wild oat plants had almost entirely shattered prior to combine harvesting. Barley seeds harvested by hand from permanent quadrats were added to those harvested by combine, and barley yield calculated in terms of grams m^{-2}.

Values of the relative time of emergence of barley and wild oat, T, in each quadrat were calculated from the cumulative totals of emerged seedlings in two steps. First, we fit a simple linear regression equation using SAS[1] (Proc GLM) to the cumulative percentage of emerged barley or wild oat seedlings as a function of the number of days after planting. Second, using the predicted regression lines, we estimated T as the difference in number of days between barley and wild oat to reach 50% seedling emergence. T was negative in value if wild oat seedlings emerged before barley seedlings. Initially, we used Richards' family of growth models (Brown and Rothery 1993) to fit the seedling emergence data. However, too few data points resulted in poor convergence of the nonlinear model fits and/or unreasonable parameter estimates. Overall, a linear regression model provided a better fit to the data. In addition to estimating T at 50% seedling emergence, we also estimated T at 20 and 80% cumulative emergence. T at 50% emergence provided the largest differences in time of emergence between barley and wild oat. Thus, we used the estimates of T at 50% seedling emergence in the model evaluations.

Candidate Models. We chose 25 a priori candidate models of varying complexity (Table 1) to fit to the field data, including models commonly used in the fields of weed science and agroecology as well as lesser known models from population ecology. We also modified components of existing models. For all models, the response variable was crop yield (Y). Explanatory variables consisted of crop density (D_c), weed density (D_w) and relative time of emergence of the crop and weed (T) although models differed in number of explanatory variables and their functional specification. The number of estimable parameters (K) varied from two to seven among models, which included K-1 structural parameters and one residual variance parameter, σ^2. The structural parameters are defined in Table 1.

In candidate models including weed density as a variable (Table 1), crop yield is assumed to decrease nonlinearly with increasing weed density. However, the relationship between crop yield and crop density differs among models. Models 1A, 2A, 3A, 4A, 5A, 6A, 7A, 8A, 9A, and 10A specify a relationship whereby increasing crop densities cause a steep increase in yield at low crop densities but result in diminishing yield returns at higher crop densities, presumably due to increasing intraspecific competition. In contrast, models 1B, 2B, 3B, 4B, 5B, 6B, 7B, 8B, 9B, and 10B specify a linear relationship between crop yield and crop density. Although a linear relationship is unrealistic biologically, preliminary analyses indicated that most crop–weed competition experiments conducted in the field, including ours, do not include a sufficient range of crop densities to detect yield losses due to intraspecific competition. Thus, a linear model often better describes the relationship between crop yield and crop density in data obtained from competition experiments conducted in the field.

Table 1. Structure of 25 a priori candidate models of crop–weed competition. The response variable (Y) is crop yield, and the explanatory variables are crop density (D_c), weed density (D_w), and the time of emergence of the crop prior to the weed (T).

Model	Model structure	No. parameters (K)[a]	Model source
1A	$Y=\frac{jD_c}{1+jD_c/Y_{\max}}\left(1-\frac{iD_w}{e^{cT}+iD_w/a}\right)$	6	Modified Jasieniuk et al. (2001)
1B	$Y=jD_c\left(1-\frac{iD_w}{e^{cT}+iD_w/a}\right)$	5	Modified Model 1A[b]
1C	$Y=\frac{R_cD_c}{1+a_cD_c}\left(1-\frac{R_wD_w}{e^{cT}+a_wD_w}\right)$	6	Modified Model 1A[b]
1D	$Y=R_cD_c\left(1-\frac{R_wD_w}{e^{cT}+a_wD_w}\right)$	5	Modified Model 1A[b]
2A	$Y=\frac{jD_c}{1+jD_c/Y_{\max}}\left(1-\frac{iD_w}{1+iD_w/a}\right)$	5	Jasieniuk et al. (2001); modified Cousens (1985a, b)
2B	$Y=jD_c\left(1-\frac{iD_w}{1+iD_w/a}\right)$	4	Modified Model 2A[b]
2C	$Y=\frac{R_cD_c}{1+a_cD_c}\left(1-\frac{R_wD_w}{1+a_wD_w}\right)$	5	Modified Model 2A[b]
2D	$Y=R_cD_c\left(1-\frac{R_wD_w}{1+a_wD_w}\right)$	4	Modified Model 2A[b]
3A	$Y=\frac{jD_c}{1+jD_c/Y_{\max}}$	3	Modified Model 1A
3C	$Y=\frac{R_cD_c}{1+a_cD_c}$	3	Modified Model 3A[b]
3B = 3D	$Y = jD_c$ and $Y = R_cD_c$	2	Modified Model 3A[b]
4A	$Y=\frac{R_cD_c}{1+a_cD_c+a_wD_w}$	4	Weiner (1982); Cousens et al. (1985b); Brown and Rothery (1993)
4B	$Y=\frac{R_cD_c}{1+a_wD_w}$	3	Modified Model 4A[b]
5A	$Y=\frac{R_cD_c}{1+a_cD_c+e^{-cT}a_wD_w}$	5	Modified Model 4A; modified Cousens et al. (1987)
5B	$Y=\frac{R_cD_c}{1+e^{-cT}a_wD_w}$	4	Modified Model 5A[b]
6A	$Y=\frac{R_cD_c}{1+a_cD_c+\frac{e^{-cT}a_wD_w}{1+bD_w}}$	6	Modified Model 5A
6B	$Y=\frac{R_cD_c}{1+\frac{e^{-cT}a_wD_w}{1+bD_w}}$	5	Modified Model 6A[b]
7A	$Y=\frac{R_cD_c}{1+a_cD_c+\frac{e^{(\theta_0+\theta_1T)}}{1+e^{(\theta_0+\theta_1T)}}a_wD_w}$	6	Modified Model 5A
7B	$Y=\frac{R_cD_c}{1+\frac{e^{(\theta_0+\theta_1T)}}{1+e^{(\theta_0+\theta_1T)}}a_wD_w}$	5	Modified Model 7A[b]
8A	$Y=\frac{R_cD_c}{1+a_cD_c+\frac{\frac{e^{(\theta_0+\theta_1T)}}{1+e^{(\theta_0+\theta_1T)}}a_wD_w}{1+bD_w}}$	7	Modified Model 7A
8B	$Y=\frac{R_cD_c}{1+\frac{\frac{e^{(\theta_0+\theta_1T)}}{1+e^{(\theta_0+\theta_1T)}}a_wD_w}{1+bD_w}}$	6	Modified Model 8A[b]
9A	$Y=\frac{R_cD_c}{1+a_cD_c}e^{iD_wcT}$	5	Modified Model 5A
9B	$Y=R_cD_ce^{iD_wcT}$	4	Modified Model 9A[b]
10A	$Y=\frac{R_cD_c}{1+a_cD_c}e^{-iD_we^{-cT}}$	5	Modified Model 9A; modified Brown and Rothery (1993)
10B	$Y=R_cD_ce^{-iD_we^{-cT}}$	4	Modified Model 10A[b]

[a] There are K-1 structural parameters and one residual variance parameter, σ^2. Structural parameters include: j, initial rate of yield increase as crop density increases from zero; $Ymax$, asymptotic maximum crop yield; i, initial rate of crop yield loss as weed density increases from zero; a, asymptotic maximum percent crop yield loss as weed density increases to its maximum; c, rate at which i decreases towards zero as T becomes large; R_c, intrinsic growth rate of the crop; R_w, intrinsic growth rate of the weed; a_c = coefficient of intraspecific competition; a_w = coefficient of interspecific competition; b, coefficient of weed intraspecific competition; and the shape parameters, θ_0 and θ_1.

[b] See Materials and Methods section for details of the model modification.

Models 1C, 2C, and 3C (Table 1) are models 1A, 2A, and 3A, respectively, that have been reformulated to make

$$a_c = \frac{j}{Y_{\max}}$$

and

$$a_w = \frac{i}{a}.$$

Parameter j estimates the initial rate of yield increase as barley density increases from zero, *Ymax* is the asymptotic maximum crop yield, i estimates the initial rate of yield loss as weed density increases from zero, and a is the asymptotic maximum percent yield loss as weed density increases to its maximum. The reformulations were necessary because we were unable to obtain maximum likelihood estimates of parameters Y_{max} and a for most data sets using the original models. The inability to obtain maximum likelihood estimates of these parameters probably resulted from the limited range of barley and wild oat densities in the data sets. Despite the estimation problems, we included the original models in our set of candidates because they are used in the field of weed science. Models 1D, 2D, and 3D are variations of models 1C, 2C, and 3C, respectively, that specify a linear relationship between crop yield and crop density. Model 3B (jD_c) is equivalent to Model 3D (R_cD_c) resulting in a total of 25 candidate models (Table 1).

Regression Analyses. We fit the 25 candidate models to the experimental data from each field location and year using the maximum likelihood estimation method of SAS (PROC NLP) with lognormal error structure. We assumed a lognormal error structure because crop yield values are positive and because residuals were not symmetrically distributed but increased with increasing expected values of barley yield.

Model Selection. We used four information criteria as measures of empirical evidence supporting each competition model relative to the others, and to identify the best model for each data set. The four information criteria were: (1) the Akaike Information Criterion, AIC (Akaike 1973); (2) a second-order small-sample version of *AIC*, namely, AIC_c (Burnham and Anderson 1998; Hurvich and Tsai 1989; Saguira 1978); (3) the Bayesian (or Schwartz's) Information Criterion, *BIC* (or *SIC*) (Schwartz 1978); and (4) the Information Complexity criterion, *ICOMP* (Bozdogan 1990, 2000; Bozdogan and Haughton 1998). Information criteria are estimators of the information lost when a model is used to approximate reality (Burnham and Anderson 1998, 2002). Thus, the model with the minimum value of *AIC*, AIC_c, *BIC*, or *ICOMP* is the model identified as "best" by each information criterion (Franklin et al. 2001; Taper 2004). To determine how much better the best model fit a data set relative to other models, we computed the difference, Δ_i, between the *AIC*, AIC_c, *BIC*, and *ICOMP* values for each model i and the minimum *AIC*, AIC_c, *BIC*, and *ICOMP* values obtained for the best model.

Results and Discussion

Model Fits to Individual Data Sets. The 25 candidate models differed in how well they described the five data sets, as measured by the four information criteria (Table 2). Clearest results were obtained for data sets from Kalispell. Three of the four information criteria, *AIC*, AIC_c, and *BIC*, indicated that models 1A and 1C best explained barley yield based on two data sets (R1 and R2) from barley–wild oat competition experiments conducted at Kalispell in 1993 (Table 2). Moreover, the data's support for models 1A and 1C was much stronger than the support for the remaining 23 models, as revealed by ΔAIC, ΔAIC_c, and ΔBIC values that greatly exceeded 10 for all remaining models, with the exception of model 10A, which had Δ_i values ranging from 5.5 to 7.8. The larger Δ_i is, the worse the model is at describing the data. As a rough rule of thumb, Δ_i values greater than 10 indicate models with essentially no support from the data, whereas Δ_i values from about 4 to 7 indicate models with substantially less support than the best model (Burnham and Anderson 1998). Strong support from the data for models 1A and 1C is indicated by Δ_i values ≤ 2. Because model 1C is simply a reformulated version of model 1A (see Materials and Methods), both of the selected models are essentially equivalent. Strong support for models 1A and 1C indicates that the three explanatory variables in the models, barley tiller density (D_c), wild oat tiller density (D_w), and the relative time of emergence of barley and wild oat (T) all influenced barley yield in Kalispell in 1993. Further, the high values of Δ_i for other models with the same three variables indicates that the mathematical relationship between barley yield and D_c, D_w, T in models 1A and 1C is better supported by the data than the relationship between the variables in other models.

In contrast to *AIC*, AIC_c, and *BIC*, the information criterion *ICOMP* selected model 3C as the best model of barley yield for Kalispell in 1993 (Table 2). Model 3C is a much simpler model than the models 1A and 1C selected by *AIC*, AIC_c, and *BIC* (Table 1). Model 3C consists of a single explanatory variable, i.e., crop density, and two estimable structural parameters. In contrast, the best models selected by *AIC*, AIC_c, and *BIC* consisted of three explanatory variables and five estimable structural parameters. The different result obtained using ICOMP reflects its larger penalty term for overparameterization relative to the other information criteria. The aim of model selection using information criteria is to choose the most parsimonious model that provides an accurate approximation of the structural information in a data set (Burnham and Anderson 1998). The measure of parsimony varies among information criteria, however. *AIC*, AIC_c, and *BIC* penalize models with greater complexity, where complexity is characterized by the number of parameters. *ICOMP* also penalizes models with greater complexity. However, *ICOMP* characterizes model complexity not only by the number of parameters but also by their redundancy and estimation instability (Bozdogan 2000; Bozdogan and Haughton 1998; J. Ferguson et al., unpublished data). Thus, the *ICOMP* penalty term includes the number of model parameters as well as the degree of interdependence among them. Parameters with highly correlated values are considered redundant. Although *ICOMP* selected model 3C as the best model for the data from Kalispell, the R1 data set also supported models 1A, 3A, and 3B = 3D, as indicated by $\Delta ICOMP$ values ≤ 2 (Table 2).

For barley–wild oat competition experiments conducted in Bozeman, *AIC*, AIC_c, and *BIC* revealed that the model best approximating the data differed between 1993 and 1994 (Table 2). In 1993, *AIC* and AIC_c selected model 1C as the best model, which was also the model most highly supported

Table 2. Summary of the fit of 25 a priori crop yield models to individual data sets from three locations in Montana. Each model's structure is shown in Table 1. K is the number of parameters in the regression model plus 1 for σ^2, n is the number of data points in the data set, Max log (L) is the maximized log-likelihood value, AIC is Akaike's Information Criterion, AIC_c is Akaike's Information Criterion with small-sample (second-order) bias adjustment, BIC is the Bayesian Information Criterion, and $ICOMP$ is Bozdogan's Information Complexity Criterion. $\Delta AIC = AIC - \min AIC$; $\Delta AIC_c = AIC_c - \min AIC_c$; $\Delta BIC = BIC - \min BIC$; $\Delta ICOMP = ICOMP - \min ICOMP$. The best approximating model for a data set is indicated by $\Delta_i = 0$ in bold type. Models with $0 < \Delta_i \leq 2$ in bold type also have substantial support from the data and should receive consideration in making inferences. Missing values (—) indicate models with at least one parameter that could not be estimated for the data set.

Location	Year	Model	K	n	Max log(L)	AIC	ΔAIC	AIC_c	ΔAIC_c	BIC	ΔBIC	$ICOMP$	$\Delta ICOMP$
Bozeman	1993	1A	6	125	105.858	—	—	—	—	—	—	—	—
		1B	5	125	118.858	—	—	—	—	—	—	—	—
		1C	6	125	122.166	−232.332	**0**	−231.620	**0**	−215.362	4.881	−185.646	34.332
		1D	5	125	120.409	−230.818	**1.514**	−230.313	**1.307**	−216.676	3.567	−207.234	12.744
		2A	5	125	102.926	—	—	—	—	—	—	—	—
		2B	4	125	117.364	—	—	—	.	—	—	—	—
		2C	5	125	120.524	−231.048	**1.284**	−230.544	**1.076**	−216.907	3.336	−189.309	30.669
		2D	4	125	119.026	−230.052	2.280	−229.719	**1.901**	−218.739	**1.504**	−205.595	14.383
		3A	3	125	95.603	—	—	—	—	—	—	—	—
		3B = 3D	2	125	95.603	−187.207	45.125	−187.109	44.511	−181.550	38.693	−187.871	32.106
		3C	3	125	105.988	−205.976	26.356	−205.778	25.842	−197.491	22.752	−193.044	26.934
		4A	4	125	117.966	−227.931	4.401	−227.598	4.022	−216.618	3.625	−198.599	21.379
		4B	3	125	117.364	−228.728	3.604	−228.529	3.091	−220.243	**0**	−215.963	4.014
		5A	5	125	119.261	−228.521	3.811	−228.017	3.603	−214.379	5.864	−201.119	18.859
		5B	4	125	118.858	−229.715	2.617	−229.382	2.238	−218.402	**1.841**	−219.270	**0.708**
		6A	6	125	121.086	−230.173	2.159	−229.461	2.159	−213.203	7.040	−190.228	29.750
		6B	5	125	119.867	−229.733	2.599	−229.229	2.391	−215.592	4.651	−207.740	12.237
		7A	6	125	119.260	−226.520	5.812	−225.808	5.812	−209.550	10.693	−181.602	38.376
		7B	5	125	118.857	−227.714	4.618	−227.210	4.410	−213.572	6.671	−200.321	19.656
		8A	7	125	121.086	−228.173	4.159	−227.215	4.405	−208.375	11.868	−157.247	62.731
		8B	6	125	119.866	−227.732	4.600	−227.020	4.600	−210.762	9.481	−191.573	28.405
		9A	5	125	106.917	−203.835	28.497	−203.330	28.290	−189.693	30.550	−180.389	39.589
		9B	4	125	98.170	−188.340	43.992	−188.007	43.613	−177.027	43.216	−158.786	61.191
		10A	5	125	120.081	−230.162	2.170	−229.657	**1.963**	−216.020	4.223	−202.989	16.989
		10B	4	125	119.388	−230.777	**1.555**	−230.444	**1.176**	−219.464	**0.779**	−219.978	**0**
Bozeman	1994	1A	6	169	222.036	—	—	—	—	—	—	—	—
		1B	5	169	215.629	—	—	—	—	—	—	—	—
		1C	6	169	222.489	−432.978	9.547	−432.460	9.546	−414.199	9.546	−382.145	24.800
		1D	5	169	216.160	−422.319	20.206	−421.951	20.055	−406.670	17.075	−392.531	14.414
		2A	5	169	220.669	—	—	—	—	—	—	—	—
		2B	4	169	214.156	—	—	—	—	—	—	—	—
		2C	5	169	220.668	−431.337	11.188	−430.969	11.037	−415.687	8.058	−383.415	23.530
		2D	4	169	214.335	−420.670	21.855	−420.426	21.580	−408.151	15.594	−391.919	15.026
		3A	3	169	151.037	—	—	—	—	—	—	—	—
		3B = 3D	2	169	151.037	−298.075	144.450	−298.002	144.004	−291.815	131.930	−297.744	109.201
		3C	3	169	157.823	−309.646	132.879	−309.500	132.506	−300.256	123.489	−295.629	111.316
		4A	4	169	220.811	−433.623	8.902	−433.379	8.627	−421.103	2.642	−398.242	8.704
		4B	3	169	214.156	−422.312	20.213	−422.166	19.840	−412.922	10.823	−405.969	**0.977**
		5A	5	169	222.934	−435.868	6.657	−435.500	6.506	−420.218	3.527	−396.899	10.046
		5B	4	169	215.629	−423.257	19.268	−423.014	18.992	−410.738	13.007	−406.195	**0.750**
		6A	6	169	222.938	−433.876	8.649	−433.358	8.648	−415.097	8.648	−380.082	26.863
		6B	5	169	216.163	−422.326	20.199	−421.957	20.049	−406.676	17.069	−392.865	14.080
		7A	6	169	227.262	−442.525	**0**	−442.006	**0**	−423.745	**0**	−404.777	**2.168**
		7B	5	169	219.351	−428.701	13.824	−428.333	13.673	−413.052	10.693	−403.447	3.498
		8A	7	169	227.266	−440.531	**1.994**	−439.836	2.170	−418.622	5.123	−388.006	18.939
		8B	6	169	216.163	−420.326	22.199	−419.808	22.198	−401.547	22.198	−360.700	46.245
		9A	5	169	163.886	−317.771	124.754	−317.403	124.603	−302.121	121.624	−293.117	113.828
		9B	4	169	160.261	−312.522	130.003	−312.278	129.728	−300.002	123.743	−288.080	118.866
		10A	5	169	221.288	−432.575	9.950	−432.207	9.799	−416.926	6.819	−395.556	11.389
		10B	4	169	216.092	−424.183	18.342	−423.940	18.066	−411.664	12.081	−406.945	**0**
Kalispell R1	1993	1A	6	68	32.582	−53.164	**0**	−51.787	**0**	−39.847	**0**	7.752	**1.377**
		1B	5	68	−1.994	13.988	67.152	14.956	66.743	25.086	64.933	37.438	31.063
		1C	6	68	32.427	−52.855	**0.309**	−51.478	**0.309**	−39.538	**0.309**	23.553	17.178
		1D	5	68	−2.019	14.037	67.201	15.005	66.792	25.135	64.982	32.656	26.281
		2A	5	68	19.200	—	—	—	—	—	—	—	—
		2B	4	68	−3.019	14.038	67.202	14.673	66.460	22.916	62.763	53.345	46.970
		2C	5	68	19.321	—	—	—	—	—	—	—	—
		2D	4	68	−2.847	13.694	66.858	14.329	66.116	22.572	62.419	26.599	20.224
		3A	3	68	8.793	−11.585	41.579	−11.210	40.577	−4.927	34.920	8.235	**1.861**
		3B = 3D	2	68	−3.019	10.038	63.202	10.223	62.010	14.477	54.324	7.492	**1.117**
		3C	3	68	8.793	−11.585	41.579	−11.210	40.577	−4.927	34.920	6.375	**0**
		4A	4	68	10.953	−13.906	39.258	−13.271	38.516	−5.028	34.819	48.437	42.062
		4B	3	68	−2.509	11.017	64.181	11.392	63.179	17.676	57.523	20.247	13.873
		5A	5	68	12.945	−15.889	37.275	−14.922	36.865	−4.792	35.055	40.062	33.688
		5B	4	68	−1.798	11.596	64.760	12.231	64.018	20.474	60.321	47.364	40.989
		6A	6	68	17.406	−22.811	30.353	−21.434	30.353	−9.494	30.353	41.260	34.885
		6B	5	68	−2.189	—	—	—	—	—	—	—	—

Table 2. Continued.

Location	Year	Model	K	n	Max log(L)	AIC	ΔAIC	AIC_c	ΔAIC_c	BIC	ΔBIC	$ICOMP$	$\Delta ICOMP$
		7A	6	68	19.304	−26.607	26.557	−25.230	26.557	−13.290	26.557	34.214	27.840
		7B	5	68	−1.798	13.596	66.760	14.564	66.351	24.694	64.541	57.035	50.660
		8A	7	68	19.293	−24.587	28.577	−22.720	29.067	−9.050	30.797	20.735	14.360
		8B	6	68	−0.568	13.136	66.300	14.513	66.300	26.453	66.300	86.911	80.536
		9A	5	68	8.794	−7.588	45.576	−6.620	45.167	3.509	43.356	30.461	24.086
		9B	4	68	−2.814	13.628	66.792	14.263	66.050	22.506	62.353	37.313	30.938
		10A	5	68	25.923	—	—	—	—	—	—	—	—
		10B	4	68	−1.814	11.627	64.791	12.262	64.049	20.506	60.353	39.262	32.888
Kalispell R2	1993	1A	6	71	39.338	−66.676	**0**	−65.363	**0**	−53.099	**0**	−13.841	5.620
		1B	5	71	10.240	−10.479	56.197	−9.556	55.807	0.834	53.933	1.372	20.832
		1C	6	71	39.338	−66.676	**0**	−65.363	**0**	−53.099	**0**	4.646	24.106
		1D	5	71	10.243	−10.487	56.189	−9.564	55.799	0.826	52.273	—	—
		2A	5	71	25.519	−41.038	25.638	−40.115	25.248	−29.725	23.374	12.690	32.150
		2B	4	71	10.245	−10.491	56.185	−9.567	55.796	0.823	53.922	9.196	28.656
		2C	5	71	25.519	−41.038	25.638	−40.115	25.248	−29.725	23.374	6.600	26.061
		2D	4	71	10.248	—	—	—	—	—	—	—	—
		3A	3	71	20.847	−35.694	30.982	−35.336	30.027	−28.906	24.193	−11.933	7.528
		3B = 3D	2	71	9.604	−15.208	51.468	−15.032	50.331	−10.683	42.416	−17.137	2.324
		3C	3	71	20.847	−35.694	30.982	−35.336	30.027	−28.906	24.193	−19.460	**0**
		4A	4	71	23.197	−38.393	28.283	−37.787	27.576	−29.342	23.757	1.170	20.630
		4B	3	71	10.531	−15.061	51.615	−14.703	50.660	−8.273	44.826	−5.430	14.031
		5A	5	71	27.007	−44.014	22.662	−43.091	22.272	−32.701	20.398	22.375	41.835
		5B	4	71	10.655	−13.310	53.366	−12.704	52.659	−4.259	48.840	−4.348	15.112
		6A	6	71	28.576	−45.151	21.525	−43.838	21.525	−31.575	21.524	17.788	37.249
		6B	5	71	10.293	−10.585	56.091	−9.662	55.701	0.728	53.827	—	—
		7A	6	71	27.011	−42.022	24.654	−40.709	24.654	−28.446	24.653	25.909	45.370
		7B	5	71	10.791	−11.583	55.093	−10.660	54.703	−0.269	52.830	16.669	36.130
		8A	7	71	24.489	—	—	—	—	—	—	—	—
		8B	6	71	11.301	—	—	—	—	—	—	—	—
		9A	5	71	20.957	−31.913	34.763	−30.990	34.373	−20.600	32.499	−10.606	8.854
		9B	4	71	10.382	−12.763	53.913	−12.157	53.206	−3.713	49.386	1.804	21.265
		10A	5	71	34.442	−58.883	7.793	−57.960	7.403	−47.570	5.529	−7.482	11.979
		10B	4	71	10.600	−13.200	53.476	−12.594	52.769	−4.150	48.949	−4.486	14.975
Moccasin	1994	1A	6	126	137.225	—	—	—	—	—	—	—	—
		1B	5	126	137.959	−265.917	**0.456**	−265.417	**0.761**	−251.736	8.867	−253.505	13.172
		1C	6	126	138.145	−264.291	2.082	−263.585	2.593	−247.273	13.330	−239.360	27.317
		1D	5	126	137.959	−265.917	**0.456**	−265.417	**0.761**	−251.736	8.867	−252.932	13.745
		2A	5	126	136.025	—	—	—	—	—	—	—	—
		2B	4	126	136.207	−264.413	**1.960**	−264.082	2.096	−253.068	7.535	−253.910	12.767
		2C	5	126	136.223	−262.446	3.927	−261.946	4.232	−248.265	12.338	−238.361	28.316
		2D	4	126	136.207	−264.413	**1.960**	−264.082	2.096	−253.068	7.535	−253.046	13.631
		3A	3	126	135.138	—	—	—	—	—	—	—	—
		3B = 3D	2	126	135.138	−266.275	**0.098**	−266.178	**0**	−260.603	**0**	−266.677	**0**
		3C	3	126	135.393	−264.787	**1.586**	−264.590	**1.588**	−256.278	4.325	−249.953	16.724
		4A	4	126	135.428	−262.855	3.518	−262.525	3.653	−251.510	9.093	−236.502	30.175
		4B	3	126	135.152	−264.305	2.068	−264.108	2.070	−255.796	4.807	−255.114	11.563
		5A	5	126	138.049	−266.097	**0.276**	−265.597	**0.581**	−251.916	8.687	−237.566	29.111
		5B	4	126	135.456	−262.912	3.461	−262.582	3.596	−251.567	9.036	−251.694	14.983
		6A	6	126	137.339	−262.677	3.696	−261.971	4.207	−245.659	14.944	−234.033	32.645
		6B	5	126	137.263	−264.526	**1.847**	−264.026	2.152	−250.344	10.259	−249.511	17.166
		7A	6	126	136.236	—	—	—	—	—	—	—	—
		7B	5	126	138.187	−266.373	**0**	−265.873	**0.305**	−252.192	8.411	−227.158	39.519
		8A	7	126	138.321	−262.642	3.731	−261.693	4.485	−242.788	17.815	−222.486	44.191
		8B	6	126	138.277	−264.553	**1.820**	−263.848	2.330	−247.536	13.067	−238.608	28.070
		9A	5	126	136.460	−262.920	3.453	−262.420	3.758	−248.738	11.865	−232.634	34.043
		9B	4	126	136.046	−264.092	2.281	−263.762	2.416	−252.747	7.856	−239.925	26.752
		10A	5	126	137.935	−265.870	**0.503**	−265.370	**0.808**	−251.688	8.915	−237.866	28.811
		10B	4	126	135.439	−262.877	3.496	−262.547	3.631	−251.532	9.071	−251.356	15.321

by the data sets from Kalispell. *BIC* selected model 4B. However, *AIC*, AIC_c, and *BIC* also indicated that several other models had substantial support from the data, as indicated by Δ_i values ≤ 2 in Table 2. In contrast, *AIC*, AIC_c, and *BIC* selected model 7A as the best model of barley yield based on the 1994 field data. Model 7A has six parameters and complex exponential terms (Table 1). In contrast, *ICOMP* selected a much simpler best model, 10B, with only four parameters, again reflecting the greater stringency of *ICOMP*. For field data from Moccasin, the results based on *AIC* and AIC_c were much less conclusive than those obtained based on *BIC* and *ICOMP* (Table 2). No one model had strong support from the data, according to *AIC* and AIC_c, rather, many were almost equally supported. In contrast, *BIC* and *ICOMP* selected model 3B = 3D as the best model. Further, the high ΔBIC and $\Delta ICOMP$ values obtained for the remaining 24 models (Table 2) indicated that no other model had strong support from the data. The best model, 3B = 3D, is the simplest model of the 25 a priori candidate models considered. Model 3B = 3D is a linear model with a single explanatory variable, crop density, and one estimable structural parameter.

Table 3. Summary of the fit of 25 a priori crop yield models to the full data set resulting from pooling individual data sets over locations and years. $\sum K$ is the number of model parameters, $\sum n$ is the number of data points, and $\sum$ Maximized $\log\left(L\left(\hat{\underline{\theta}}\hat{\sigma}^2 \middle| data\right)\right)$ is the maximized log-likelihood value, summed across the individual data sets. $\Delta_i = 0$ indicates the best model for the pooled data. Only models in which all parameters could be estimated were included in the analysis. Missing values (—) indicate excluded models.

Model	$\sum K$	$\sum n$	$\sum$ Maximized $\log\left(L\left(\hat{\underline{\theta}}\hat{\sigma}^2 \middle| data\right)\right)$	ΔAIC	ΔAIC_c	ΔBIC	$\Delta ICOMP$
1A	30	559	537.0	—	—	—	—
1B	25	559	480.69	—	—	—	—
1C	30	559	554.57	**0**	**0**	**0**	245.45
1D	25	559	482.75	133.64	132.54	112.00	192.17
2A	25	559	504.34	—	—	—	—
2B	20	559	474.95	—	—	—	—
2C	25	559	522.26	—	—	—	—
2D	20	559	476.97	—	—	—	—
3A	15	559	411.42	—	—	—	—
3B = 3D	10	559	388.36	292.42	289.28	205.88	99.95
3C	15	559	428.84	221.46	218.81	156.55	144.84
4A	20	559	508.35	72.44	70.46	29.16	358.237
4B	15	559	474.69	129.76	127.11	64.85	**0**
5A	25	559	520.20	58.74	57.66	37.11	294.55
5B	20	559	478.80	131.54	129.57	88.27	145.46
6A	30	559	527.34	54.46	54.44	54.44	281.75
6B	25	559	481.40	—	—	—	—
7A	30	559	529.07	—	—	—	—
7B	25	559	485.39	128.36	127.27	106.73	251.19
8A	35	559	530.46	—	—	—	—
8B	30	559	485.04	—	—	—	—
9A	25	559	437.01	225.12	224.02	203.47	260.47
9B	20	559	402.05	285.04	283.08	241.78	233.26
10A	25	559	539.67	—	—	—	—
10B	20	559	479.71	129.72	127.76	86.46	106.37

Model Fits to the Pooled Data Set. Consistent with the results obtained for individual data sets, *AIC*, AIC_c, and *BIC* selected a more complex model than *ICOMP* for data pooled over locations and years (Table 3). *AIC*, AIC_c, and *BIC* selected model 1C as the model of barley yield best supported by the pooled data. Model 1C also best explained barley yield in the individual data sets from Kalispell and Bozeman in 1993. Large values of Δ_i for the other models (Table 3) indicate that D_c, D_w, T, and their functional relationship with barley yield in model 1C are more strongly supported by the pooled data than the variables and their functional specification in the remaining candidate models.

As observed for individual data sets, *ICOMP* selected a much simpler model, 4B, as the best approximation of the information in the pooled data. Model 4B consists of two explanatory variables, D_c and D_w, and two structural parameters, R_c and a_w. The model describes a linear relationship between barley yield and barley density. Selection of this model as the best supported by the data indicates that the pooled data provide no evidence of intraspecific competition affecting barley yield. The pooled data did provide evidence of interspecific competition, however, because model 4B specifies a negative relationship between crop yield and weed density. Relative time of emergence of barley and wild oat was not a variable in the selected model.

Variation Among Sites and Years. As observed for barley in this study and other crops in previous studies, crop–weed interference relationships often vary among sites and years (e.g., Fischer et al. 2004; Jasieniuk et al. 1999, 2001; Lindquist et al. 1999). This is not surprising, given the numerous factors and processes that affect crop yield including environmental variables such as soil moisture and/or fertility, which can vary from year to year and site to site, as well as biological processes, such as intraspecific and interspecific competition. A major strength of the model selection approach using information–theoretic criteria is that simultaneous comparison of many models allows more straightforward identification of the variables and processes affecting crop yield at a site in a particular year than the conventional hypothesis testing approach. In this study, for instance, identification of models 1A and 1C by the 1993 data from Kalispell indicated that increasing densities of both barley and wild oat, as well as the relative time of emergence of two species, influenced barley yields. The selected models pointed to intraspecific and interspecific competition as important processes determining barley yield in Kalispell. In contrast, data from Moccasin in 1994 identified a linear model with a single variable, crop density, and one estimable structural parameter. Strong support for a linear model over all others indicates that the Moccasin data provided no evidence of intraspecific or interspecific competition and suggests that abiotic, rather than biotic, factors determined barley yield in Moccasin in 1994. Although data from only one year were available for each location, support of the different best models for each location corresponds well with differences in average annual precipitation and barley yields between the two sites. The northwestern region of Montana, where Kalispell is located, is the wettest region of the state and has high average barley yields, whereas the central region, where Moccasin is located, is the driest with correspondingly lower yields (http://plantsciences.montana.edu/Crops/barley). It is not surprising, therefore, that biotic factors played a greater role in determining barley yields in Kalispell, whereas abiotic factors are likely to have been more important in Moccasin.

Table 4. Crop–weed competition models strongly supported by data from field experiments conducted at three locations in Montana. Strong support from the data is indicated by Δ_i values ≤ 2 in Tables 2 and 3. The best model for a data set, indicated by $\Delta_i = 0$, is listed first and in bold type.

Data Set	ΔAIC	ΔAIC_C	ΔBIC	$\Delta ICOMP$
Bozeman 1993	**1C**	**1C**	**4B**	**10B**
	1D	1D	2D	5B
	2C	2C	5B	
	10B	2D	10B	
		10A		
		10B		
Bozeman 1994	**7A**	**7A**	**7A**	**10B**
	8A			4B
				5B
Kalispell R1 1993	**1A**	**1A**	**1A**	**3C**
	1C	1C	1C	1A
				3A
				3B
Kalispell R2 1993	**1A**	**1A**	**1A**	**3C**
	1C	**1C**	**1C**	
Moccasin 1994	**3B/7B**	**3B**	**3B**	**3B**
	1B	1B		
	1D	1D		
	2B	3C		
	2D	5A		
	3C	7B		
	5A	10A		
	6B			
	8B			
	10A			
Pooled data set	**1C**	**1C**	**1C**	**4B**

Variation among Information Statistics. For all data sets, including the pooled data, *AIC* and AIC_c generally gave similar results and selected the same best models. Because sample sizes were large relative to the number of estimated parameters in each model, the small-sample bias correction term in AIC_c did not influence choice of the best model, although the number of models with support from the data differed for two data sets. *BIC* selected the same best model as *AIC* and AIC_c for four of the five individual data sets as well as the pooled data. For the remaining data set (Bozeman 1993), *BIC* selected a model of lower order (dimension) than *AIC* and AIC_c. *BIC* also differed from *AIC* and AIC_c by indicating that the Moccasin data only supports one model, i.e., model 3B (Table 4), which is the simplest model in the set of candidate models (Table 1). *AIC* and AIC_c are generally known to overparameterize data and select models of the maximum order allowed (Bozdogan 1987; Taper 2004). In contrast, BIC generally selects models of lower order than *AIC* and AIC_c, as was the case here (Table 4), and does not tend to overparameterize. Of the four information criteria used in this study, *ICOMP* generally selected the simplest best model for each data set. Because *ICOMP*'s penalty term takes into account not only the number of model parameters but also the degree of interdependence among them (Bozdogan 2000; Bozdogan and Haughton 1998; J. Ferguson et al., unpublished data), models of lower order are generally selected by *ICOMP* in contrast to *AIC*, AIC_c, or *BIC*, as discussed previously.

Selection of a Barley Yield Model. Whether to use *AIC*, AIC_c, *BIC*, or *ICOMP* as the criterion for model selection depends on the purpose for which models are used (Cox 1990; Taper 2004). In weed management bioeconomic models, crop–weed competition equations are primarily used to predict economic thresholds. For minimizing prediction errors, *AIC* or AIC_c often can be an appropriate selection criterion despite its tendency to overparameterize. In this study, *BIC*, *AIC*, and AIC_c all selected the same model (1C) for the pooled data and most of the individual data sets (Table 4). Thus, based on the data and models analyzed here, our results strongly point to model 1C as the best yield equation to use in a barley–wild oat bioeconomic model. In contrast to *BIC*, *AIC*, and AIC_c, the information criterion *ICOMP* selected several different best models for the individual data sets as well as a unique one for the pooled data, thus making it difficult to confidently choose a yield model using this information criterion. Although yield model 1C is clearly the best candidate for a bioeconomic model based on the consensus of three of the four information criteria and the data used in this study, it might not remain the best model as more experimental data become available, including data from more years and sites and a wider range of barley and wild oat densities. However, an additional strength of the information–theoretic approach is that it readily lends itself to the addition and comparison of more datasets and models as they become available.

Strengths of Model Selection Using an Information–Theoretic Approach. Model selection using information–theoretic criteria shows promise for crop–weed competition modeling. The approach allows simultaneous assessment of multiple crop yield or yield loss models, which are ranked and quantitatively assessed in terms of the evidence from experimental data for one model over another. By simultaneously evaluating the empirical support for multiple competing models, the information–theoretic approach significantly reduces the risk of model misidentification and poor model performance (Burnham and Anderson 1998, 2002; Johnson and Omland 2004; Taper 2004). Moreover, the set of candidate models can be reevaluated in a straightforward manner as additional crop–weed competition data sets and models become available. Selection of equations for the prediction of crop yields or yield losses is central to the development of bioeconomic models for weed management. Information–theoretic statistics offer a rigorous, objective method for identifying models for such weed management decision tools.

Sources of Materials

[1] SAS System for Windows V8, SAS Institute Inc., Cary, NC 27513.

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